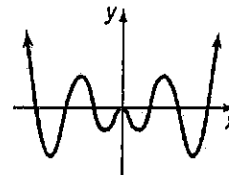


4 CHAPTER 3 Polynomial and Rational Functions

The illustration shows the graph of a polynomial function.

- Is the degree of the polynomial even or odd?
- Is the leading coefficient positive or negative?
- Is the function even, odd, or neither?
- Why is x^2 necessarily a factor of the polynomial?
- What is the minimum degree of the polynomial?
- Formulate five different polynomials whose graphs could look like the one shown. Compare yours to those

of other students. What similarities do you see? What differences?



Chapter Test

- Graph $f(x) = (x - 3)^4 - 2$.
- $f(x) = 3x^2 - 12x + 4$
 - Determine if the function has a maximum or a minimum and explain how you know.
 - Algebraically, determine the vertex of the graph.
 - Determine the axis of symmetry of the graph.
 - Algebraically, determine the intercepts of the graph.
 - Graph the function by hand.
- For the polynomial function $g(x) = 2x^3 + 5x^2 - 28x - 15$,
 - Determine the maximum number of real zeros that the function may have.
 - Find bounds to the zeros of the function.
 - List the potential rational zeros.
 - Determine the real zeros of g . Factor g over the reals.
- Find the complex zeros of $f(x) = x^3 - 4x^2 + 25x - 100$.
- Solve $3x^3 + 2x - 1 = 8x^2 - 4$ in the complex number system.

Problems 6 and 7, find the domain of each function. Find any horizontal, vertical, or oblique asymptotes.

$$6) g(x) = \frac{2x^2 - 14x + 24}{x^2 + 6x - 40}$$

$$7) r(x) = \frac{x^2 + 2x - 3}{x + 1}$$

- Sketch the graph of the function in Problem 7. Label all intercepts, vertical asymptotes, horizontal asymptotes, and oblique asymptotes. Verify your results using a graphing utility.

In Problems 9 and 10, write a function that meets the given conditions

- Fourth-degree polynomial with real coefficients; zeros: $-2, 0, 3 + i$
- Rational function; asymptotes: $y = 2, x = 4$; domain: $\{x \mid x \neq 4, x \neq 9\}$
- Use the Intermediate Value Theorem to show that the function $f(x) = -2x^2 - 3x + 8$ has at least one real zero on the interval $[0, 4]$.

In Problems 12 and 13, solve each inequality algebraically. Write the solution set in interval notation. Verify your results using a graphing utility.

$$12. 3x^2 - x - 4 \geq x^2 - 3x + 8$$

$$13. \frac{x + 2}{x - 3} < 2$$

- Given the polynomial function $f(x) = -2(x - 1)^2(x + 2)$, do the following:
 - Find the x - and y -intercepts of the graph of f .
 - Determine whether the graph crosses or touches the x -axis at each x -intercept.
 - Find the power function that the graph of f resembles for large values of $|x|$.
 - Using results of parts (a)–(c), describe what the graph of f should look like. Verify this with a graphing utility.
 - Approximate the turning points of f rounded to two decimal places.
 - By hand, use the information in parts (a)–(e) to draw a complete graph of f .

The table gives the average home game attendance A for the St. Louis Cardinals (in thousands) during the years 1995–2003, where x is the number of years since 1995.

SOURCE: www.baseball-almanac.com

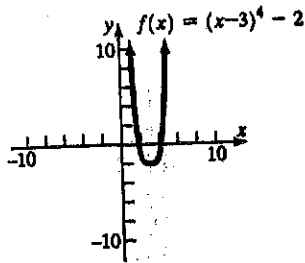
x	0	1	2	3	4	5	6	7	8
A	24.570	32.774	32.519	39.453	40.197	41.191	38.390	37.182	35.930

- Use a graphing utility to find the quadratic function of best fit to the data. Round coefficients to three decimal places.
- Use the function found in part (a) to estimate the average home game attendance for the St. Louis Cardinals in 2004.

130. (a) even (b) positive (c) even (d) The graph touches the x -axis at $x = 0$, but does not cross it there. (e) 8

Chapter Test (page 244)

1.



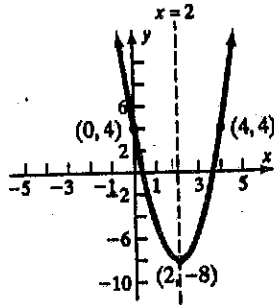
2. (a) The leading coefficient (i.e., the coefficient on x^2) is positive so the graph will open up. Thus, the graph has a minimum.

(b) $(2, -8)$

(c) $x = 2$

(d) $(0, 4), \left(\frac{6 - 2\sqrt{6}}{3}, 0\right), \left(\frac{6 + 2\sqrt{6}}{3}, 0\right)$.

(e)



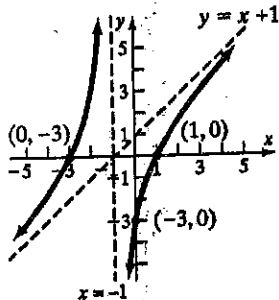
3. (a) 3

(b) Every zero of g lies between -15 and 15 .

(c) $\frac{p}{q}: \pm\frac{1}{2}, \pm 1, \pm\frac{3}{2}, \pm\frac{5}{2}, \pm 3, \pm 5, \pm\frac{15}{2}, \pm 15$

(d) $-5, -\frac{1}{2}, 3; g(x) = (x + 5)(2x + 1)(x - 3)$

8.



4. $4, -5i, 5i$

5. $\left\{1, \frac{5 - \sqrt{61}}{6}, \frac{5 + \sqrt{61}}{6}\right\}$

6. Domain: $\{x | x \neq -10, x \neq 4\}$
Asymptotes: $x = -10, y = 2$

7. Domain: $\{x | x \neq -1\}$;
Asymptotes: $x = -1, y = x + 1$

9. Answers may vary. One possibility is $f(x) = x^4 - 4x^3 - 2x^2 + 20x$

10. Answers may vary. One possibility is $r(x) = \frac{2(x-9)}{(x-4)(x-9)}$

11. $f(0) = 8; f(4) = -36$

Since $f(0) = 8 > 0$ and $f(4) = -36 < 0$, the Intermediate Theorem guarantees that there is at least one real:

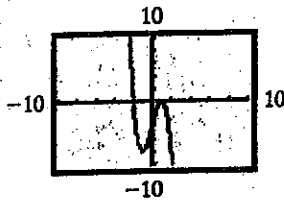
12. $\{x | -\infty < x \leq -3 \text{ or } 2 \leq x < \infty\}$, or $(-\infty, -3] \cup [2, \infty)$

13. $\{x | x < 3 \text{ or } x > 8\}$, or $(-\infty, 3) \cup (8, \infty)$

14. (a) $(0, -4), (-2, 0), (1, 0)$

Because the power function has an odd degree, we expect the ends of the graph to go in opposite directions. The leading coefficient is negative, so the left side of the graph will go up and the right side will go down.

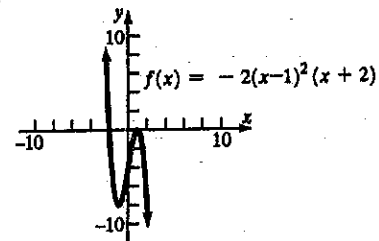
Reading the graph from left to right, we would expect to see it decreasing and cross the x -axis at $x = -2$. Somewhere between $x = -2$ and $x = 1$ the graph turns so that it is increasing, touches the x -axis at $x = 1$, turns around and decreases from that point on.



(b) Touches at 1 and crosses at -2 (c) $y = -2x^3$

(e) $(-1, -8)$, and $(1, 0)$.

(f)



15. (a) $A(x) = -0.604x^2 + 6.038x + 25.350$

(b) The model predicts that the average 2004 home game attendance for the St. Louis Cardinals will be 30,768.